DISCUSSION

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A NOTE ON ALEKSEEV'S ARTICLE "THE NATURE OF THE RANQUE EFFECT"

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From the Editors

Alekseev's article "The Nature of the Ranque Effect" was published in issue No. 4 of 1964 of this journal. In publishing the article, the Editors' intention was not only to present the method proposed by the author for designing vortex separating chambers, but also to draw the attention of readers to this urgent question.

Some readers comments on Alekseev's article are given below. The Editors consider that further discussion would not be profitable, since the question is now clear enough. It is hoped that this airing of the problem will have positive results.

Readers' Comments

V. M. Brodyanskii and A. V. Martynov:

In his article "The Nature of the Ranque Effect," Alekseev proposes "a new hypothesis and an elementary theory of the Ranque effect." In our opinion, this article occupies a special position amongst published work on the vortex effect, since all the author's concepts differ fundamentally from those of others [1-6].

This prompts us to make a brief analysis of the author's views. Since Alekseev presents his material unsysmatically and his article contains numerous repetitions, we have organized our review under the three main heads: physical picture of the vortex process; proposed mathematical formulas; comparison of theoretical and test data.

1. The process of expansion of a gas in a vortex tube and the corresponding representation on a T-S diagram are given incorrectly.

The author writes (p. 126): "...the over-all process of decrease of temperature is divided into two stages. Firstly, a throttling process takes place along the line AB (i = const) from p_{00} to P_{012}."

In fact, the gas first expands in the nozzle adiabatically in the ideal case with S = const from the initial pressure p_1 to the pressure beyond the nozzle p_2 [6].

Later, Alekseev discusses the process as follows: "...an adiabatic expansion occurs in the tube (compression of the peripheral and expansion of the axial gas layers as a result of centrifugal forces) along the line BD (S = const)," "...under the influence of centrifugal forces the peripheral layers of gas are compressed and therefore heated, whereas the axial layers expand and therefore are cooled" (p. 126, 122).

These statements are incorrect. In the first place, there is no compression of gas in the peripheral layers. On the contrary, the pressure in the gas flowing from the nozzle into the peripheral layers drops continuously from p_2 to the pressure of the cold flow, in proportion to the motion in the tube [7, 8]. Secondly, in assuming a process of expansion in the tube with S = const, the author is making at the very outset, an assumption that is incorrect in principle. The condition S = const is not fulfilled in the motion of viscous fluids with internal friction and nonuniform velocity field (and therefore, in particular, not in a vortex tube).

The law of isentropic change of state of a gas (p/ρ^k = const) is valid only for flows in which there are no friction forces [4], and it is not appropriate for the analysis of rotational flows of a perfect gas (in the aerodynamic sense) with a nonuniform velocity field.

In explaining the heating of the outer layers by the action of centrifugal forces ("...the centrifugal forces create a static temperature gradient along the tube radius"), the author does not take into account the important fact that in the active region (under real operating conditions) exactly the opposite picture is observed — the layers near the wall have a lower thermodynamic temperature than those on the axis [8]. This phenomenon fundamentally contradicts the theory of centrifugal forces. The explanation of the vortex process should not be reduced to the action of centrifugal forces. The vortex process results from a whole series of closely related phenomena [4, 5], and in the active region of the tube the effect of turbulent energy transfer between the axial and wall flows is decisive.

Finally, Alekseev makes a serious error in asserting that the axial velocities of the gas particles are small compared with the peripheral velocities (p. 121), and in citing Martynovskii and Voitko on this point. The reference is unsound, since, in fact, these authors particularly stress that the axial velocities are commensurate with the tangential and must not be neglected (p. 84) [9].

2. The above erroneous statements lead Alekseev to the incorrect mathematical formulas which he proposes for calculation of the vortex effect. Let us examine, for example, formulas (3) and (8), which describe the variation of static pressure p and temperature T along the radius.

On the basis of the formula for variation of the stagnation temperature along the radius [3]

$$\frac{i_0}{i_{01}} = 1 - (1 - \bar{r}^2)\xi^2 \cos^2\alpha = 1 - (1 - \bar{r}^2)\frac{w_1^2}{w_{max}^2} \cos^2\alpha, \tag{1}$$

where the subscript "1" refers to parameters at $\bar{r} = 1$ (when $r = r_1$).

It can be shown that the static temperature for an equilibrium vortex process in a section of the nozzle is constant [2-5]. Substituting $i_0 = i + w_\tau^2/2$, into (1), we obtain

$$\frac{i}{i_{01}} = 1 - (1 - \bar{r}^2)\frac{w_1^2}{w_{max}^2}\cos^2\alpha - \frac{w_\tau^2}{2i_{01}}. \tag{2}$$

From the law of rotation $w_\tau/r = \text{const}$ we have

$$\frac{i}{i_{01}} = 1 - (1 - \bar{r}^2)\frac{w_1^2}{w_{max}^2}\cos^2\alpha - \frac{r^2}{r_1^2}\frac{w_{\tau_1}^2}{w_{max}^2},$$

or

$$\frac{i}{i_{01}} = 1 - (1 - \bar{r}^2)\frac{w_1^2}{w_{max}^2}\cos^2\alpha - \bar{r}^2\frac{w_1^2\cos^2\alpha}{w_{max}^2},$$

$$\frac{i}{i_{01}} \simeq \frac{T}{T_{01}} = 1 - \frac{w_1^2}{w_{max}^2}\cos^2\alpha = 1 - \xi^2\cos^2\alpha. \tag{3}$$

It is clear from (3) that the static temperature does not depend on the radius, i.e., is constant over the section.

To find the distribution of static pressure along the radius, we must use the Euler equation in cylindrical coordinates [3]

$$\frac{1}{\rho}\frac{dp}{dr} = \frac{w_\tau^2}{r}. \tag{4}$$

Expressing the density in terms of the speed of sound a ($\rho = kp/a^2 = kp/(k-1)i$), and substituting in (4), with account for the value of i from (3) and $w_\tau^2 = r^2A^2$, we obtain

$$\frac{dp}{p} = \frac{k}{k-1}\frac{r^2A^2}{i_{01}(1 - \xi^2\cos^2\alpha)}\frac{dr}{r}. \tag{5}$$

Integrating (5), we have

$$\ln p = \frac{k}{k-1}\frac{A^2}{i_{01}(1 - \xi^2\cos^2\alpha)}\frac{r^2}{2} + B. \tag{6}$$

Determining the constant of integration for the condition $r = r_1$ and transforming (6), we obtain

$$p = \exp\left[\frac{k}{(k-1)i_{01}(1 - \xi^2\cos^2\alpha)}\frac{w_{\tau_1}}{2}\left(\frac{r^2}{r_1^2} - 1\right) + \ln p_1\right]. \tag{7}$$

Substituting $w_{\tau_1}^2 = w_1^2\cos^2\alpha$ and $2i_{01} = w_{max}^2$, we finally have

$$p = \exp\left[\frac{k}{(k-1)(1-\xi^2\cos^2\alpha)}\xi^2\cos^2\alpha(\bar{r}^2-1)+\ln p_1\right]. \quad (8)$$

Thus, the static pressure in a section of the tube varies according to an exponential law.

Alekseev's formulas (3) and (8) are therefore incorrect.

Formulas (9) and (10), containing double integrals, give in fact the trivial result:

$$T_0 = T + Aw^2/2gC_p.$$

As the main conclusion of his theory, the author gives the relation $\mu = \psi^2$ (p. 125): "...the square of the ratio of the diameter of the aperture in the vortex chamber to that of the tube is equal to the cold-flow fraction by weight." It is clear from an examination of the boundary conditions $\mu = 0$ and $\mu = 1$, that this relation is incorrect. Thus, when $\mu = 0$ ($\psi^2 = 0$), according to Eq. (14), given by the author, $p = p_1$, i.e., there is no pressure gradient along the tube radius. This contradicts both the experimental data of [6] and theory, which clearly indicates that when $\mu = 0$ in rotational flows there is a gradient of static pressure.

3. The author makes incorrect comparisons of calculated and test values. An example is his comparison of static pressure data (Table 1). A calculation made for various $\mu = \psi^2$ is compared with the test data of [7], obtained for the single value $\mu = 0$. The same is also true of temperature (Table 3). The test data presented in the table relate only to $\mu = 0.5$.

Thus, examination of Alekseev's article shows that the author's erroneous ideas about the vortex process, together with his unsuitable mathematical relations and comparative test data, cannot form a basis for the verification of the new "theory" he proposes and, indeed, are evidence of the complete unsoundness of his "new hypothesis."

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A. P. Merkulov:

Having studied Alekseev's article "The Nature of the Ranque Effect," I should like to make some critical comments.

The assumption made by the author in his initial remarks that the angular velocity is constant along the entire radius of the vortex basically contradicts the work cited [6], in which a region of increasing velocity with decrease of radius was clearly observed at the periphery of the vortex.

With the radial displacement of particles assumed by the author (even at negligibly small radial velocities) the law of conservation of moment of momentum will act to create a potential flow with radial velocity distribution $\omega r^2 = \text{const}$. Only subsequent action of viscous forces can transform this distribution into rotation with $\omega = \text{const}$.

Therefore there is no foundation for the author's supposition that angular velocity is constant over the entire radius of the vortex.

In determining the mean stagnation temperature of the cooled flow (formula (10) on p. 124), the author averages over the area of the section, which, when there are large radial static pressure gradients, leads to an appreciable radial density gradient for an adiabatic distribution law.

It is therefore necessary to find the total mean mass temperature from the expression

$$T_{0x} = \int_0^{r_x} T_* \rho r dr / \int_0^{r_x} \rho r dr.$$

The same change must be made in (11). This averaging gives a higher value of T_{0x} than that calculated by the author, since the hotter layers at greater radii have greater density.

If a core of radius r_x flows into the surrounding medium through the orifice of a diaphragm, then layers adjacent to r_x will emerge with greater velocity than layers near the axis, due to the static pressure gradient in the vortex, and, since the former are also hotter, the mean mass temperature of the cooled flow beyond the diaphragm will be even higher, i. e., appreciably higher than the experimental value.

The analysis given by the author presupposes that separation in the vortex is completely realized, but, with correct averaging, the value of the cooling effect should be less than that obtained experimentally (Table 2), which indicates the imperfection of the hypothesis advanced.

Expression (20) does not correspond to the experimental data, since, at $r_x = 0.45$, experiment gives $\mu_{0pt} = 0.3$, and at $r_x = 0.55$, $\mu_{0pt} = 0.65$, these values depending significantly on the total flow rate through the vortex tube.

It is unlikely that Alekseev's hypothesis could explain the reverse vortex tube, in which heated gas flows through the orifice in the diaphragm, and cooled gas through the throttle at the "hot end."

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